$$\beta_{13}=\frac{3}{4}+\frac{1}{4\pi i}\ln\frac{1-\nu}{3+\nu},\qquad \beta_3=\frac{1}{2}+\frac{1}{2\pi i}\ln\frac{1-\nu}{3+\nu}$$

Calculation of the determinant $D(\beta)$ of system (6.5) by using /9/ yields the following values for $\nu=0.3$, which ensure that the system is solvable $D(\beta_{11})\approx 0.0694$, $D(\beta_{12})\approx 0.5876$, $D(\beta_3)\approx 0.1498$, $D(\beta_{13})\approx -0.3359+0.3283i$.

The presence of an imaginary part in β_{13} and β_3 shows that the contact forces in the last two problems contain oscillatory factors in addition to non-integrable singularities.

REFERENCES

1. ONISHCHUK O.V. and POPOV G.YA., On certain problems of the bending of plates with cracks and thin inclusions. Izv. Akad. Nauk SSSR, Mekhan. Tverd. Tela, 4, 1980.
2. POPOV G.YA., Elastic Stress Concentration Around Stamps. Slits, Thin Inclusions, and Reinforcements. Nauka, Moscow, 1982.
3. ONISHCHUK O.V. and POPOV G.YA., Non-integrable solutions in plate bending problems (the case of a delaminating inclusion). Thirteenth National Conf. on the Theory of Plates and Shells, Pt. 4, 54-59, Izd. Tallin Politekhn. Inst. Tallin, 1983.
4. ONISHCHUK O.V., POPOV G.YA. and PROTSEROV YU.S., On certain contact problems for reinforced plates. PMM, 48, 2, 1984.
5. GEL'FAND I.M. and SHILOV G.E., Generalized Functions and Operations on Them. Fizmatgiz, Moscow, 1958.
6. GAKHOV F.D., Boundary Value Problems, Nauka, Moscow, 1977.
7. DWIGHT G.W., Tables of Integrals and Other Mathematical Formulas, Nauka, Moscow, 1977.
8. MIKHLIN S.G., Integral Equations and Their Application to Certain Problems of Mechanics, Mathematical Physics, and Engineering. Gostekhizdat, Moscow-Leningrad, 1949.
9. Tables of the Logarithmic Derivative of the Gamma Function and Its Derivatives in the Complex Domain. Vychisl. Tsentr Akad. Nauk SSSR, Moscow, 1965.

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ON THE INVERSE PROBLEM OF THE SCATTERING OF ELASTIC WAVES BY A THIN FOREIGN INCLUSION*

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The problem of the remote determination of the shape of an isolated scatterer is considered using longitudinal elastic waves. It is assumed that the scatterer is a thin elastic solid of revolution situated in an elastic space under conditions of rigid contact and that Poisson's ratios of the medium and the scatterer are the same. The use of multifrequency wave information is a special feature of the solution of the problem. The problems of the uniqueness and stability of the solution obtained are also studied.

We mean by the inverse scattering problem the problem of determining the form of the scattering region by analysing the scattered field. The problem, as a rule, is one of a number of ill-posed problems of mathematical physics /1/. The current interest in developments in this direction is caused by the practical needs experienced in such fields as acoustic diagnostics, geophysics, hydroacoustics, medicine, etc. At present several approaches to the study of the form of closed isolated scatterers are known /2—4/. Here the corresponding direct problem of mathematical physics was formulated as a boundary value problem for the Helmholtz equation with Dirichlet boundary conditions and Newmann or impedance boundary conditions on the unknown surface of the body whose location was being determined.

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The present paper uses the method given in /5/ to solve the problem of determining the form of a thin elastic solid of revolution using the experimentally known scattering amplitude of longitudinal displacement waves specified on a discrete set of the sounding frequencies in a fixed direction in space. It is assumed here that Poisson's ratios of the medium and the solid in question are the same, and the solid is in rigid contact with the elastic medium.

1. Let a space filled with an isotropic elastic medium with Lamé coefficients λ, μ of density ρ, contains a foreign inclusion D_*, bounded by a smooth closed surface S and characterized by the Lamé coefficients λ_1, μ_1 and density ρ_1, in rigid contact with the medium. We assume that a plane longitudinal displacement wave arriving from infinity impinges on the body D_*

$$\mathbf{u}^n(\mathbf{x}) = A_0\mathbf{l}\exp[i\omega_L(\mathbf{l}, \mathbf{x})] \tag{1.1}$$

Here and henceforth A_0 is the amplitude of the incoming wave, $\mathbf{l} = (l_1, l_2, l_3)$ is its direction of propagation, $(\cdot, \cdot)$ denotes a scalar product, c_T, c_L are the rates of propagation of the transverse and longitudinal waves, ω is the frequency, $\mathbf{x} = (x_1, x_2, x_3)$ is the radius vector of any point of the space originating in D_*, the time multiplier $\exp(-i\omega\tau)$ is omitted, and $\omega_A = \omega c_A^{-1}$, $A = L, T$.

If $\mathbf{u}^p(\mathbf{x})$ is the scattered field displacement vector, then the problem of determining the field $\mathbf{u}(\mathbf{x})$ with components $u_j(\mathbf{x}) = u_j^n(\mathbf{x}) + u_j^p(\mathbf{x})$ $(j = 1, 2, 3)$ under the condition $\lambda_1\mu = \lambda\mu_1$, is equivalent to solving the functional Eq.(6)

$$\varkappa^{-1}(\mathbf{x})\mathbf{u}(\mathbf{x}) = \mathbf{u}^n(\mathbf{x}) + Q^\omega[\mathbf{u}](\mathbf{x}), \quad \mathbf{x} \in R^3 \tag{1.2}$$

$$Q^\omega[\mathbf{u}](\mathbf{x}) = \delta_1\rho\omega^2\int_D u_m(\mathbf{y})\boldsymbol{\Gamma}_m(\mathbf{x}, \mathbf{y})\,d\mathbf{y} +$$
$$\delta_2\int_S u_i(\mathbf{y})\,n_j(\mathbf{y})\left\{\lambda\frac{\partial\boldsymbol{\Gamma}_m(\mathbf{x}, \mathbf{y})}{\partial y_m}\delta_{ij} + \mu\left(\frac{\partial\boldsymbol{\Gamma}_i(\mathbf{x}, \mathbf{y})}{\partial y_i} + \frac{\partial\boldsymbol{\Gamma}_j(\mathbf{x}, \mathbf{y})}{\partial y_i}\right)\right\}dS_y$$
$$\varkappa^{-1}(\mathbf{x}) = \begin{cases} 1, & \mathbf{x} \in R^3\setminus D_* \\ 1 = 2^{-1}\delta_2, & \mathbf{x} \in S \\ 1 - \delta_2, & \mathbf{x} \in D \end{cases}$$

where dS_y is the area element of the surface S at the point $\mathbf{y} = (y_1, y_2, y_3)$, $\mathbf{n} = (n_1, n_2, n_3)$ is the direction of the external normal to S, $\delta_{ij}(i, j = 1, 2, 3)$ is the Kronecker delta, $\boldsymbol{\Gamma}_i$ is a vector with components Γ_{1i}, Γ_{2i}, Γ_{3i}; and repeated indices denote summation. We also have

$$\Gamma_{ij}(\mathbf{x}, \mathbf{y}) = \frac{\exp[i\omega_T|\mathbf{x} - \mathbf{y}|]}{4\pi\mu|\mathbf{x} - \mathbf{y}|}\delta_{ij} -$$
$$\frac{(\rho\omega^2)^{-1}}{4\pi}\frac{\partial^2}{\partial x_i\partial x_j}\frac{\exp[i\omega_L|\mathbf{x} - \mathbf{y}|] - \exp[i\omega_T|\mathbf{x} - \mathbf{y}|]}{|\mathbf{x} - \mathbf{y}|}$$
$$\delta_1 = \rho_1\rho^{-1} - \mu_1\mu^{-1}, \ \delta_2 = 1 - \mu_1\mu^{-1}, \quad d\mathbf{y} = dy_1dy_2dy_3$$

and the field $\mathbf{u}^p(\mathbf{x})$ satisfies the Sommerfeld radiation condition. We shall write this condition in the form

$$\mathbf{u}^p(\mathbf{x}) = -(4\pi R)^{-1}\sum_{A=L,T}\exp[i\omega_A R]\,\mathbf{f}^A(\omega; \mathbf{l}, \mathbf{v}) + O(R^{-2}) \tag{1.3}$$
$$(R = |x| \to \infty)$$

where $\mathbf{f}^L$, $\mathbf{f}^T$ are the scattering amplitudes of the longitudinal and transverse waves and $\mathbf{v} = \mathbf{x}\,|\mathbf{x}|^{-1}$ is the direction of the vector $\mathbf{x}$.

Then from (1.2), (1.3) we obtain

$$\mathbf{f}^L(\omega; \mathbf{l}, \mathbf{v}) = -\mathbf{v}\left\{\delta_1\omega_L^2\int_D(\mathbf{v}, \mathbf{u})\exp[-i\omega_L(\mathbf{v}, \mathbf{y})]\,d\mathbf{y} - \right. \tag{1.4}$$
$$\left.\frac{i\omega_L}{\lambda + 2\mu}\delta_2\int_S[(\mathbf{u}, \mathbf{n})\lambda + 2\mu(\mathbf{v}, \mathbf{n})(\mathbf{v}, \mathbf{u})]\exp[-i\omega_L(\mathbf{v}, \mathbf{y})]\,dS_y\right\}$$
$$\mathbf{f}^T(\omega; \mathbf{l}, \mathbf{v}) = -(\boldsymbol{\delta}_j - \mathbf{v}v_j)\left\{\delta_1\omega_T^2\int_D u_j\exp[-i\omega_T(\mathbf{v}, \mathbf{y})]\,d\mathbf{y} - \right. \tag{1.5}$$
$$\left. i\omega_T\delta_2\int_S[(\mathbf{n}, \mathbf{v})u_j + n_j(\mathbf{u}, \mathbf{v})]\exp[-i\omega_T(\mathbf{v}, \mathbf{y})]\,dS_y\right\} \quad (j = 1, 2, 3)$$

where δ_j is a vector with components $\delta_{j1}, \delta_{j2}, \delta_{j3}$.

The question now arises, how, using the experimentally known scattering amplitude of the longitudinal and transverse waves specified in a fixed spatial direction at a discrete set of frequencies, we can determine the surface S, while possessing some a priori information about its shape.

Mathematically, the inverse problem in question reduces to that of determining the surface S from the systems of functional equations obtained from (1.2), (1.4) or (1.2), (1.5) with $\omega = \omega_1, \omega_2, \ldots, \omega_N$. The latter problem cannot, in general, be solved exactly. Any attempt to solve it approximately must be based on some restrictions imposed on the properties of the scatterer and on the frequence range of the sounding field. The latter is dictated by the need to have an approximate solution of the direct problem, i.e. an approximate solution of (1.2), available.

It can be shown that if we have the inequality

$$\frac{\omega_T^2}{4\pi|2-\delta_2|}\left\{|\delta_1|\max_{x\in D_*}\int_D \frac{dy}{|x-y|} + |\delta_2|(2\xi^{-2}+\xi^2)\int_S dS\right\} \leqslant M < 1 \tag{1.6}$$

$$(\xi = c_T c_L^{-1})$$

where M is a constant, then a solution of (1.2) with $x \in D_*$ exists in the Banach space $C(D_*)$ of functions continuous on D_*, is unique, and can be represented by a uniformly convergent Neumann series.

Indeed, let r, r_1 and r_2 be the spectral radii of the operators $\varkappa(x) Q^\omega$ $\varkappa(x)[Q^\omega - Q]$ and $\varkappa(x) Q$, $x \in S$ where Q is the value of the operator Q^ω when $\omega = 0$, $x \in S$. Using the mean value theorem we find, by direct computation, that

$$\|Q^\omega - Q\| = \sup_{\substack{u\in C(D_*)\\ u\neq 0}} \frac{\|(Q^\omega - Q)[u](x)\|_C}{\|u\|_C} \leqslant \frac{\omega_T^2}{8\pi}\left[|\delta_1| \times \right.$$

$$\left.\max_{x\in D_*}\int_D \frac{dy}{|x-y|} + |\delta_2|(2\xi^{-2}+\xi^2)\int_S dS\right] = M_1, \quad \|u\|_C = \max_{y\in D_*}|u(y)|$$

and from this we find that $r_1 \leqslant \varkappa(x) M_1$, $x \in \dot{S}$.

The operator $2(2-\delta_2)^{-1}Q$ was investigated in /6/ where it was shown that its spectral radius is not greater than unity. Taking into account the boundedness of the operator Q on $C(S)$, we obtain $r_2 < 1$, $x \in S$. Since $r \leqslant r_1 + r_2$, putting $M = 1 - r_2$ we arrive at the assertion made above, since in this case the series

$$\sum_{n=0}^{\infty} [\varkappa(x) Q^\omega]^n, \quad x \in D_*$$

converges absolutely.

We note that the restriction $\lambda_1\mu = \lambda\mu_1$ is not essential, since when $\lambda_1\mu \neq \lambda\mu_1$, the problem reduces to a sequence of problems for Eq.(1.2) with $x \in D_*$ with the unrestricted recurrent terms belonging to the space $C(D_*)$ /6/. However, in this case the expressions for the scattering amplitudes become much more complex.

In particular, the zeroth approximation is given by the equation

$$u_0(x) = \varkappa(x) u^n(x), \quad x \in D_*$$

and we write the first terms of the Neumann series for the scattering amplitude (1.4), (1.5) in the form

$$f_1^A(\omega; l, v) = 2A_0\omega_A^2\beta^A \int_D \exp[i\omega_L(\eta^A, y)]\, dy, \quad A = L, T \tag{1.7}$$

$$\beta^L = v\left\{-\frac{\delta_1}{2(1-\delta_2)}(v, l) + \frac{\delta_2}{2-\delta_2}(l, \eta^L)[2\xi^2(1+(v,l)) - 1]\right\}$$

$$\beta^T = -[l - v(v, l)]\left\{\frac{\delta_1}{2(1-\delta_2)} + \frac{\delta_2}{2-\delta_2}[2\xi(v, l) - 1]\right\}$$

$$\eta^L = l - v, \quad \eta^T = \xi l - v$$

When $\omega \to 0$, we obtain from (1.7)

$$f_1^A \simeq f^A \simeq 2A_0\omega_A^2\beta^A V$$

where V is the volume of the scatterer.

Inequality (1.6) in fact forces us to consider the inverse problem in the low-frequency domain in which, generally speaking, it is not easy to ensure that the method of solution given below is powerful enough. We shall therefore assume that inequality (1.6) holds, not because the wave number is small, but because of the other characteristic parameters of the problem. Namely, let the equation of the surface S be written in cylindrical coordinates r, φ, t in the form

$$r = \varepsilon F(t, \varphi), \quad 0 \leqslant t \leqslant a, \quad 0 \leqslant \varphi \leqslant 2\pi$$

where the function $F(t, \varphi)$ is assumed to be sufficiently smooth on the surface of the unit cylinder $G = \{0 \leqslant t \leqslant a,\ 0 \leqslant \varphi \leqslant 2\pi,\ r = 1\}$, $\varepsilon > 0$ is a small parameter.

Then we have, from (1.7), the following expression for the principal terms of the uniform asymptotic representation of the scattering amplitudes in terms of the parameter $\omega_A a\varepsilon$:

$$\mathbf{f}^A(\omega; \mathbf{l}, \mathbf{v}) = A_0 \beta^A (\omega_A \varepsilon)^2 \int_G F^2(t, \varphi) \exp[i\omega_A \eta_3{}^A t]\, dt\, d\varphi \qquad (A = L, T) \tag{1.8}$$

We also have the following estimate for the surfaces of class C^2:

$$|\mathbf{f}^A - \mathbf{f}_\varepsilon{}^A| \leqslant \varepsilon^{1/2} (\omega_A a\varepsilon)^2 [c_1(1 + c_2\omega_A) + c_3\omega_A \varepsilon^{1/2}] \tag{1.9}$$

where $c_i\ (i = 1, 2, 3)$ are constants independent of ω and ε.

The estimate (1.9) can be proved directly from the inequalities

$$|\mathbf{f}^A - \mathbf{f}_\varepsilon{}^A| \leqslant |\mathbf{f}^A - \mathbf{f}_1{}^A| + |\mathbf{f}_1{}^A - \mathbf{f}_\varepsilon{}^A|$$
$$|\mathbf{f}^A - \mathbf{f}_1{}^A| \leqslant c_1\omega_A \max_{\mathbf{x}\in S} \left|\left(\frac{\delta_2}{2} I^* + Q^\omega\right)[\mathbf{u}^n](\mathbf{x})\right| \left(\int_S dS\right)^{1/2} V^{1/2}$$

where I is the unit operator, and subsequent application of the mean-value theorem.

Relations (1.8) represent the starting relations for formulating the inverse problem consisting of determining the function $\varepsilon F(t, \varphi)$.

2. Let the region G and the orientation of the surface S relating to the vector $\mathbf{l}$ be both known. Further, let the function $\mathbf{f}^L(\omega; \mathbf{l}, \mathbf{v})$ or $\mathbf{f}^T(\omega; \mathbf{l}, \mathbf{v})$ be known in a fixed direction in space, at a discrete set of frequencies $\omega = \omega_1, \omega_2, \ldots, \omega_N$, such that inequality (1.6) is not violated. Then we obtain from (1.8) a system of integral Fredholm equations of the first kind for the function $[\varepsilon F(t, \varphi)]^2$

$$f^A(\omega_m; \mathbf{l}, \mathbf{v}) = A_0 \beta^A (k_m \varepsilon)^2 \int_G F^2(t, \varphi) \exp[ik_m \eta_3{}^A t]\, dt\, d\varphi \tag{2.1}$$
$$(k_m = \omega_m c_A^{-1}, \qquad m = 1, 2, \ldots, N)$$

where the index A is fixed.

Analysing relations (1.8) we find that the direction of the vector $\mathbf{f}^L$ is the same as that of the vector $\mathbf{v}$, and the vector $\mathbf{f}^T$ is orthogonal to $\mathbf{f}^L$. Then, if the vector $\mathbf{v}$ tracks a unit sphere, the vector $\mathbf{f}^T$ will form a smooth vector field tangent to the sphere and will vanish at least in the directions $\mathbf{v} = \pm \mathbf{l}$. Therefore the field $\mathbf{f}^L$ will predominate in a direction close to the direction of backward scattering. Since this domain of angles of observation is the most preferable one from the practical point of view, we shall henceforth assume that $A = L$ and omit the index L.

Now let the vectors $\mathbf{l}$ and $\mathbf{v}$ be fixed, so that $\eta_3 \neq 0$. We shall assume without loss of generality that $\beta_3 \neq 0$. Then from (2.1) it follows that in order to determine the function $\varepsilon F(t, \varphi)$ in terms of $f_3(\omega_m; \mathbf{l}, \mathbf{v})$ $(m = 1, 2, \ldots, N, \ldots)$ uniquely, we must introduce the a priori information $F(t, \varphi) = F(t)$, since we cannot determine uniquely a function of two variables in terms of a function of a single variable.

Then from (2.1) we obtain

$$\int_0^a p(t) \exp(i\alpha_m t)\, dt = \gamma_m, \quad m = 1, 2 \ldots, N, \ldots \tag{2.2}$$
$$\alpha_m = k_m \eta_3, \quad p(t) = [\varepsilon F(t)]^2, \ \gamma_m = f_3(\omega_m; \mathbf{l}, \mathbf{v})(2\pi A_0 k_m{}^2)^{-1}$$

It can be shown /4/ that if the numbers α_m $(m = 1, 2, \ldots, N;\ N \to \infty)$ for a set of condensation point at zero, i.e. if the corresponding wave lengths have a condensation point a infinity, then the solution of system (2.2) is unique in the space $L_2[0, a]$ and its normal pseudosolution in this space can be written, for any finite N, in the form

$$p_N(t) = a^{-1} \sum_{n=1}^N A_n \exp(-i\alpha_n t) \tag{2.3}$$

with the coefficients A_n, satisfying the linear algebraic system

$$\sum_{n=1}^N A_n \alpha_{nm} = \gamma_m, \qquad m = 1, 2, \ldots, N \tag{2.4}$$
$$(\alpha_{nm} = \{\exp[ia(\alpha_m - \alpha_n)] - 1\}[ia(\alpha_m - \alpha_n)]^{-1})$$

Let $G(g_1, \ldots, g_N)(g_n = \exp(i\alpha_n t),\ n = 1, 2, \ldots, N)$ be the determinant of the system (2.4). Since $G(g_1, \ldots, g_N)$ is the Gram determinant and system $g_1, g_2, \ldots, g_N$ is linearly independent, we have /7/

$$0<G(g_1,\ldots,g_N)\leqslant\prod_{n=1}^{N}\alpha_{nn}=1$$

Here $G(g_1,\ldots,g_N)=1$ if and only if $\alpha_{nm}=0$ $(n\neq m,\ n,m=1,2,\ldots,N)$. Therefore, the solution of system (2.4) is unique and the quantity $G(g_1,\ldots,g_N)$ can be arbitrarily small when $N\to\infty$. This implies that the solution of the system of integral Eqs.(2.2), as well as the solution of system (2.4), both need to be regularized when $N\to\infty$. With this in mind, we shall consider a family of operators R_N, depending on an integer-valued parameter and defined as follows: $R_N\mathbf{f}=p_N$, whose p_N is given by (2.3) and $\mathbf{f}=(f_1,\ldots,f_N)$ $(f_n=f_3(\omega_n;\ \mathbf{l},\ \mathbf{v}),\ n=1,\ldots,N)$. Since /4/ $\|p-p_N\|_{L_2}\to 0$ and the solution of system (2.4) is unique, it follows that the family of linear operators R_N is the regularizing set for the equations (2.2) /8/. Since the components of the vector $\mathbf{f}$ are obtained by observation, it follows that the quantities $\gamma_n\ (n=1,2,\ldots,N,\ldots)$ will contain a certain amount of noise, with the additive components δ_n distributed according to one rule or another. In order for the solution (2.3) to be stable for small variations in the initial data $\mathbf{f}_\delta$, the number N taken must be matched to the level of error within which the components $\delta_n\ (n=1,2,\ldots,N,\ldots)$ are specified.

By virtue of the triangle inequality we have

$$\|p-R_N\mathbf{f}_\delta\|_{L_2}\leqslant\|p_N-R_N\mathbf{f}_\delta\|_{L_2}+\|p-p_N\|_{L_2} \tag{2.5}$$

and

$$\|p_N-R_N\mathbf{f}_\delta\|_{L_2}^2=G^{-2}(g_1,\ldots,g_N)\sum_{i,j=1}^{N}\Big(\sum_{n,m=1}^{N}\alpha_{nm}\Delta_{in}\overline{\Delta}_{jm}\Big)\delta_i\overline{\delta}_j\leqslant$$
$$\delta^2G^{-2}(g_1,\ldots,g_N)\Big(\sum_{n=1}^{N}\Big(\sum_{i=1}^{N}|\Delta_{in}|^2\Big)^{1/2}\Big)^2=\delta^2B^2(N)$$
$$\delta^2\geqslant\sum_{n=1}^{\infty}\delta_n\overline{\delta}_n$$

Here Δ_{nm} is the cofactor of the element α_{nm} of the determinant $G(g_1,\ldots,g_N)$, and the bar denotes the complex conjugate.

Since the second term on the right-hand side of inequality (2.5) tends to zero as $N\to\infty$, by choosing $N=N(\delta)$ so that $\delta B(N(\delta))\to 0$ as $\delta\to 0$ and $N\to\infty$, we can obtain /1, 8/ $R_{N(\delta)}\mathbf{f}_\delta\to p(t)$ in the metric of the space $L_2[0,a]$. This condition is satisfied, for example, by the number $N(\delta)=\max\{N: B(N)\leqslant c\delta^{-q}\}$, where $c>0,\ 0<q<1$.

Thus the use of a priori information concerning the properties of the scatterer introduced above, enables an algorithm to be compiled for obtaining an approximate solution of the inverse problem in question, and also enables its Tikhonov correctness to be investigated /8/.

REFERENCES

1. TIKHONOV A.N. and ARSENIN V.YA., Methods of Solving Ill-posed Problems. Moscow, Nauka, 1979.
2. SVESHNIKOV A.G., IL'INSKII A.S., YEREMIN YU.A and CHIVILEV A.V., Some aspects of the investigation of the problem of reestablishing the form of a perfect scatterer. Vychisl. tsentra MGU, 36, 1982.
3. MAJDA A., The inverse problem for convex bodies. Proc. Natl. Acad. Sci. USA, 73, 5, 1976.
4. YEMETS V.F., Solution of the inverse scattering problem in a linearized formulation. Zh. vychisl. matem. i matem. fiz., 24, 4, 1984.
5. YETMETS V.F., The inverse probelm of acoustic-wave scattering for thin acoustically rigid bodies. PMM 48, 1, 1984.
6. KUPRADZE V.D., Potential Methods in the Theory of Elasticity. Moscow, Fizmatgiz, 1963.
7. AKHIEZER N.I. Lectures on Approximation Theory. Mosocw, Nauka, 1965.
8. LAVRENT'EV M.M., ROMANOV V.G. and SHISHATSKII S.P., Ill-posed Problems of Mathematical Physics and Analysis. Moscow, Nauka, 1980.

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